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INTEGER SOLUTIONS OF HOMOGENEOUS BI-QUADRATIC EQUATION WITH FIVE UNKNOWNS $x^4-y^4=17\left(z^2-w^2\right)p^2$

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ABSTRACT

The homogeneous biquadratic equation represented by $x^4-y^4=17\left(z^2-w^2\right)p^2$ is considered and analysed for its non-zero distinct integer solutions. It is observed that by introducing the linear transformations $x=u+v$, $y=u-v$, $z=2u+v$, $w=2u-v$ in the given equation, one obtains infinitely many non-zero distinct integer solutions.

Keywords: *Homogeneous bi-quadratic, bi-quadratic with five unknowns, integer solutions.*

I. INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, bi-quadratic Diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-5]. In this context, one may refer [6-12] for various problems on the biquadratic Diophantine equations with four variables. In [13-16] bi-quadratic Diophantine equations with five variables are considered for integer solutions. However, often we come across homogeneous biquadratic equations and as such one may require its integral solution in its most general form. This paper concerns with the problem of determining non-zero integral solutions of the homogeneous equation with five unknowns given by $x^4-y^4=17\left(z^2-w^2\right)p^2$.

II. METHOD OF ANALYSIS

The biquadratic homogeneous equation to be solved is

$$x^4-y^4=17\left(z^2-w^2\right)p^2 \qquad (1)$$

Introduction of the linear transformations

$$x=u+v,\ y=u-v,\ z=2u+v,\ w=2u-v \qquad (2)$$

in (1) gives

$$u^2+v^2=17p^2 \qquad (3)$$

We present below different methods of solving (3) and thus, different sets of non-zero distinct solutions to (1) are obtained.

2.1 Method: 1

Let $p=a^2+b^2$ (4)

Write 17 as

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$$17 = (4+i)(4-i) \qquad (5)$$

Using (4) and (5) in (3) and employing the method of factorization, define

$$u + iv = (4+i)(a+ib)^2 \qquad (6)$$

Equating the real and imaginary parts in (6), we get

$$u = 4a^2 - 4b^2 - 2ab$$

$$v = a^2 - b^2 + 8ab$$

Substituting the values of u and v in (2), it is seen that

$$\left.\begin{aligned} x &= 5a^2 - 5b^2 + 6ab \\ y &= 3a^2 - 3b^2 - 10ab \\ z &= 9a^2 - 9b^2 + 4ab \\ w &= 7a^2 - 7b^2 - 12ab \end{aligned}\right\} \qquad (7)$$

Thus, (4) and (7) represent the distinct integer solutions to (1)

Note: 1

It is observed that 17 may also be written as

$$17 = (1+4i)(1-4i)$$

For this choice, the corresponding integer solutions to (1) are given by

$$x = 5a^2 - 5b^2 - 6ab$$

$$y = -3a^2 + 3b^2 - 10ab$$

$$z = 6a^2 - 6b^2 - 14ab$$

$$w = -2a^2 + 2b^2 - 18ab$$

$$p = a^2 + b^2$$

2.2 Method: 2

Note that (3) is also written as

$$u^2 + v^2 = 17p^2 * 1 \qquad (8)$$

Write 1 as

$$1 = \frac{(3+4i)(3-4i)}{25} \qquad (9)$$

Using (4), (5) and (9) in (8) and employing the method of factorization, define

$$u + iv = \frac{1}{5}(4+i)(3+4i)(a+ib)^2 \qquad (10)$$

Equating the real and imaginary parts in (10) and replacing a by $5A$, b by $5B$, we have

$$u = u(A,B) = 40A^2 - 40B^2 - 190AB$$

$$v = v(A,B) = 95A^2 - 95B^2 + 80AB$$

In this case, the corresponding integer solutions to (1) are found to be

$$x = x(A,B) = 135A^2 - 135B^2 - 110AB$$

$$y = y(A,B) = -55A^2 + 55B^2 - 270AB$$

$$z = z(A,B) = 175A^2 - 175B^2 - 300AB$$

$$w = w(A,B) = -15A^2 + 15B^2 - 460AB$$

$$p = p(A,B) = 25A^2 + 25B^2$$

Note: 2

It is to be noted that 1 may also be written as

$$1=\frac{(4+3i)\,(4-3i)}{25}$$

For this choice, the corresponding integer solutions to (1) are given by

$$x=x(A,B)=145A^2-145B^2-30AB$$
$$y=y(A,B)=-15A^2+15B^2-290AB$$
$$z=z(A,B)=210A^2-210B^2-190AB$$
$$w=w(A,B)=50A^2-50B^2-450AB$$
$$p=p(A,B)=25A^2+25B^2$$

Note: 3

It is worth mentioning that two more sets of integer solutions to (1) are determined by considering

(i) $17=(1+4i)(1-4i), 1=\frac{(3+4i)\,(3-4i)}{25}$

(ii) $17=(1+4i)(1-4i), 1=\frac{(4+3i)\,(4-3i)}{25}$

Remark:

In general, one may represent 1 as either

$$1=\frac{\left[2mn+i\left(m^2-n^2\right)\right]\left[2mn-i\left(m^2-n^2\right)\right]}{\left(m^2+n^2\right)^2}$$

or

$$1=\frac{\left[\left(m^2-n^2\right)+i2mn\right]\left[\left(m^2-n^2\right)-i2mn\right]}{\left(m^2+n^2\right)^2}$$

2.3 Method: 3

Observe that (3) is written in the form of ratio as

$$\frac{u+p}{4p+v}=\frac{4p-v}{u-p}=\frac{\alpha}{\beta} \qquad (11)$$

which is equivalent to the system of double equation

$$\beta u-\alpha v+p(\beta-4\alpha)=0$$
$$\alpha u+\beta v-p(\alpha+4\beta)=0$$

Applying the method of cross multiplication, we have

$$u=\left(\alpha^2-\beta^2\right)+8\alpha\beta$$
$$v=2\alpha\beta-4\left(\alpha^2-\beta^2\right)$$
$$p=\alpha^2+\beta^2 \qquad (12)$$

In view of (2), we have

$$\left.\begin{aligned}x&=10\alpha\beta-3\left(\alpha^2-\beta^2\right)\\ y&=6\alpha\beta+5\left(\alpha^2-\beta^2\right)\\ z&=18\alpha\beta-2\left(\alpha^2-\beta^2\right)\\ w&=14\alpha\beta+6\left(\alpha^2-\beta^2\right)\end{aligned}\right\} \qquad (13)$$

Thus, (12) and (13) represent the integer solutions to (1)

Note: 4

Also, (3) is written in the form of ratio as

$$\frac{u+p}{4p-v}=\frac{4p+v}{u-p}=\frac{\alpha}{\beta}$$

The corresponding integer solutions to (1) are given by

$$x=-6\alpha\beta+5\left(-\alpha^2+\beta^2\right)$$
$$y=-10\alpha\beta-3\left(-\alpha^2+\beta^2\right)$$
$$z=-14\alpha\beta+6\left(-\alpha^2+\beta^2\right)$$
$$w=-18\alpha\beta-2\left(-\alpha^2+\beta^2\right)$$
$$p=\alpha^2+\beta^2$$

III. CONCLUSION

In this paper, we have made an attempt to find infinitely many distinct integer solutions to the homogeneous biquadratic equation with five unknowns given by $x^4-y^4=17\left(z^2-w^2\right)p^2$. It is worth to note that one may also introduce the transformations $x=u+v$, $y=u-v$, $z=2uv+1$, $w=2uv-1$ and $x=u+v$, $y=u-v$, $z=uv+2$, $w=uv-2$ giving some more sets of infinitely many integer solutions to the given biquadratic equation.

To conclude, one may search for other choices of solutions to the considered biquadratic equation with five unknowns.

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